7-6 Rational Zero Theorem

Solve, given that -4 is a zero.
$x^3 - 11x + 20 = 0$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -11 & 20 \\ & & -4 & 16 & -20 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$x^2 - 4x + 5$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$\frac{4 \pm 2i}{2}$$

$\{-4, 2 \pm i\}$

Rational Zero Theorem-- $\frac{p}{q}$ is a possible zero, where:

- p $\in$ set of integral factors of the constant
- q $\in$ set of the integral factors of the leading coefficient

Solve
$0 = 6x^3 + 7x^2 - 9x + 2$

$p \in \{\pm 1, \pm 2\}$

$q \in \{\pm 1\ \pm 2\ \pm 3\ \pm 6\}$

$\frac{p}{q} \in \{\pm 1\ \pm \frac{1}{2}\ \pm \frac{1}{3}\ \pm \frac{1}{6}\ \pm 2\ \pm \frac{2}{3},$

$$\begin{array}{r|rrrr} -2 & 6 & 7 & -9 & 2 \\ & & -12 & 10 & -2 \\ \hline & 6 & -5 & 1 & \vdots\, 0 \end{array}$$

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$6x^2-5x+1$

$\underbrace{6x^2-2x}-3x+1$

$2x(3x-1)-1(3x-1)$

$(2x-1)(3x-1)=0$

$x=\frac{1}{2}$ $x=\frac{1}{3}$ $x=-2$

$\{\frac{1}{2}, \frac{1}{3}, -2\}$

Solve

$x^4 - x^3 + 7x^2 -9x - 18= 0$

$p \in \{\pm 1\ \pm 2\ \pm 3\ \pm 6\ \pm 9\ \pm 18\}$

$q \in \{\pm 1\}$

$\frac{p}{q} \in \{$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 7 & -9 & -18 \\ & & -1 & 2 & -9 & 18 \\ \hline 2 & 1 & -2 & 9 & -18 & \vdots\, 0 \\ & & 2 & 0 & 18 & \\ \hline & 1 & 0 & 9 & \vdots\, 0 & \end{array}$$

$x^2+9=0$

$x^2=-9$, $x=\pm 3i$

$\{-1, 2, \pm 3i\}$

Do:

$0 = 2x^4 + 3x^3 + 6x^2 + 12x - 8$

Solve.
$0 = x^4 - 4x^3 + 6x^2 - 8x + 8$

$p \in \{\pm 1 \ \pm 2 \ \pm 4 \ \pm 8\}$

$q \in \{\pm 1\}$

$\frac{p}{q} \in$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 6 & -8 & 8 \\ & & 2 & -4 & 4 & -8 \\ \hline 2 & 1 & -2 & 2 & -4 & \vdots\, 0 \\ & & 2 & 0 & 4 & \\ \hline & 1 & 0 & 2 & \vdots\, 0 & \end{array}$$

$\{2, 2, \pm i\sqrt{2}\}$ $\quad x^2 + 2 = 0$

Constant = 0

Solve.
$0 = x^4 - 6x^3 + 8x^2 - 48x$

$x(x^3 - 6x^2 + 8x - 48) = 0$

$p \in \{\pm 1 \ \pm 2 \ \pm 3 \ \pm 4 \ \pm 6 \ \pm 8 \ \pm 12 \ \pm 16 \ \pm 24 \ \pm 48$

$q \in \{\pm 1\}$

$\{0, 6, \pm 2i\sqrt{2}\}$

HW
p381
13, 15, 19, 25, 28, 30